Plumes and jets are naturally and frequently occurring transport phenomena arising in a variety of settings, ranging from dry convecting motion on hot days through explosive volcanic eruptions, for example, the 1915 eruption of Lassen Peak shown in Figure 1. The fluid dynamical purpose of a plume is a dynamic equilibration of a localized unstable distortion of the fluid density, which results in a vertical, coherent, motion of a parcel of fluid seeking an equilibrium density. Viscosity couples and draws fluid along with the parcel on its voyage (turbulent entrainment). If the parcel is miscible with the ambient fluid, turbulent mixing will result, accelerating the equilibration process.

A further complication is that ambient fluids typically develop stable stratifications in which the fluid density is higher at the bottom. Such is the case with the full atmosphere, whose density drops to zero in outer space, and the steady state density profile is merely the thermodynamic response of an air layer under gravitational compression. Equally interesting stratification processes occur with much sharper gradients in convective boundary layers, and in the thermoclines found in lakes and oceans. In these situations, stable transitions from high density bottom layer fluid to low density upper layer fluid may occur across a very sharp, nearly interfacial layer. Typical stratifying agents include localized high temperature gradients and/or concentration gradients (salt), and the modifications introduced by such layers can be both naturally dramatic, and socio-economically challenging. The discharge of pollutants into the atmosphere, lakes, and oceans is frequently accompanied by trapping phenomena directly attributed to the formation of such stable density layers (thermal inversions), in which the discharged
pollutants are confined away from mixing flows, and may lead to hazardous air and water quality.

Figure 1: May 22, 1915 Eruption of Lassen Peak, taken 50 miles away in Anderson, California, by Photographer, R. I. Meyers, (Thanks to Cari Kreshak and Lassen Volcanic National Park for providing the high quality image.)

**Plume Mixing and Entrainment: Modifying the large scale observables.** A light plume of fluid in a constant density environment is expected to rise continually until the Archimedian buoyancy force is reduced through the mixing of the plume with the ambient to levels at which viscous balances occur. The complete evolution requires, at minimum, the solution of the Navier-Stokes equations, with an evolving density anomaly (the plume) allowed to mix with the ambient. The mixing is turbulent, and the computational simulations are both difficult, and unavoidable to making first principles predictions. Modelers have turned to alternative, somewhat ad-hoc, yet nonetheless fundamental attempts to describe the evolution with fewer degrees of freedom than the complete fluid equations. As discussed below,
the pioneering work of Morton, Turner, and Taylor [Morton, et al. (1956)] utilized an entrainment hypothesis with a single entrainment coefficient in an attempt to describe jet (plume) profiles with reduced, nonlinear ordinary differential equations (involving only a few degrees of freedom).

The plume dynamics in stratified fluids are dramatically different [Morton, et al. (1956), Morton (1967), Turner (1995)]. In such a situation, the buoyancy of a plume of light fluid is strongly height dependent, and an initially (low altitude) light fluid parcel may well rise to a height at which a buoyancy reversal occurs and the parcel becomes neutrally buoyant. Such a situation was originally noted by Morton, et al. to cause an arrestment vertical jets of light fluid. Their experiments and modeling for a fluid with a linear stratification (linearly decreasing density with increasing height) indeed show vertical jets arresting. The limiting implications for functioning smokestacks, modeling of volcanic plumes [Sparks (1997)], and the mixing of oceanic pollutants [Fischer, et al. (1979)] is clear.

When the density transitions sharply between two distinct values, one finds mixing between low density jet (or plume) fluid and the ambient fluid, which may dramatically affect the large scale observables. Experiments, performed in the UNC Applied Mathematics Fluid Lab, further exhibit the need for improved modeling that is specifically designed to better understand turbulent mixing and entrainment.

Figure 2 shows two vertical jets, fired at approximately the same volumetric flow rate (roughly .2 g/m) and into two identically stratified fluid tanks with a prepared sharp transition from 1.06 g/cc at the bottom to 1.015 g/cc using varying salt concentrations, with transition of approximately 1in thickness, centered around the 14 inch tick on the tape. The left jet fluid is a red gauge oil, with density 0.8 g/cc (lighter than all ambient tank fluid). Recall that oil and water do not mix. The right jet fluid is a dyed alcohol-water mixture, with density 0.8 g/cc, also (initially) lighter than everything in the tank. In this case, the alcohol-water mixture may mix with the ambient fluid.

Observe the striking difference in large scale observables: The non-mixing case penetrates clear to the free surface, whereas the mixing case does not penetrate, but forms, at altitudes in the vicinity of the sharp density transition—a cloud. The alcohol jet, fired in non-stratified cases of either 1.06 g/cc or 1.015 g/cc constant density tanks will reach the free surface at these flow rates, and does not form a cloud, which demonstrates the powerful effect that an ambient sharp stratification can have upon plume dynamics, and the effect of the turbulent mixing and entrainment. There has been considerable
work on developing plume models for studying the types of behavior shown with the alcohol jet following the original work of Morton, Turner, and Taylor [Morton, et al. (1956), Sparks (1997), Caulfield & Woods(1998)], and some attempts have been directed at the multi-phase aspects of the oil jet example [Asaeda & Imberger(1993), Socolofsky, et al. (2001)]. A successful and complete modeling approach handling a full range of cases in which the mixing properties between jet fluid and ambient fluid may be continuously varied is an open challenge.

Figure 2: Vertical buoyant jets through a strong stable density step: Left is oil (0.8 g/cc), right is alcohol-water mixture (0.8 g/cc). (Thanks to former UNC Undergraduates Ryan McCabe and Daniel Healion for assistance with the experiment.)

Dynamic Plumes and Solid Wall interactions: Transient Levitation of Falling Bodies. As an extreme example in which the injected quantity cannot mix with the ambient fluid, consider recently obtained experimental results concerning the motion of falling bodies through stratified fluids (similar to tank setup in Figure 2) [Abaid, et al. (in press)]. This study has focussed upon the effect of self-generated plumes upon the falling body, and has documented situations in which a falling body may generate a dynamic plume that, through hydrodynamic coupling, may temporarily arrest the body. Of course, any body moving through a fluid experiences a hydrodynamic drag (which sets terminal velocities of falling bodies) in which the viscous boundary condition of vanishing fluid flow at the solid boundary nec-
essarily drags a blob of ambient fluid along the moving body. In a constant density fluid, there is no potential energy cost associated with moving such a parcel of ambient fluid vertically. However, in strongly stratified fluids, a parcel of fluid moved from one altitude to another may develop a potential energy (buoyancy), as when the body falls through a sharp density transition layer. The momentum of the attached blob of fluid thrusts it into the lower (heavier) fluid, at which point the blob becomes a density anomaly, and rises sharply. This motion in turn drags the falling body along with it.

Figure 3: Top: Digital snapshots of bead position on uniform 1.5 sec intervals, Middle: uniformly spaced on 0.1 sec intervals, Bottom: shadowgraph depicting the dynamic plume on same time interval as middle row [Abaid, et al. (in press)]. (Thanks to David Adalsteinsson for help with formatting the collage in his DataTank program and thanks for former UNC undergraduate Nicole Abaid for assistance with the experimental effort.)

Figure 3 shows three montages of a descending sphere at uniformly spaced times. The (5 mm radius) sphere in this case has a density of 1.04 g/cc and is falling in a stratified tank whose top is fresh water (0.997 g/cc), and whose bottom is salt water (1.039 g/cc), again with a transition thickness of approximately 1 inch. The top montage demonstrates the arrest and transient rise of the initially falling bead, and subsequent return to slow
descent, each image uniformly spaced 1.5 sec apart. The bead ultimately come to rest at the tank bottom. The middle montage is the same as the top, only uniformly spaced on 0.1 sec intervals. The lower montage is the same time sequence as the middle row, only focusing upon the shadow on the back of the tank, which highlights the entrained, plume forming fluid.

The nature of this phenomenon is both nonlinear and dynamic. The nonlinear effect of such plumes upon the motion of solid bodies has been incorporated in a reduced system of ordinary differential equations in which the drag law for the falling body is modified to account for the dynamics of the plume which may modify the relative velocity of the falling sphere [Abaid, et al. (in press)]. To describe the detailed dynamics of such transient plumes is quite difficult. Historically, there is has been more success in the modeling of plume geometries under steady state geometries. In pioneering work, Morton, Turner, and Taylor [Morton, et al. (1956), Morton (1967), Turner (1995)] were the first to model maintained plumes using an entrainment hypothesis which has become a standard in many fields [Fischer, et al. (1979), Sparks (1997)].

Steady plume models in stratified environments. In 1956, Morton, Turner, and Taylor introduced what has become the standard maintained plume models for the shape of jet plumes (plumes emanating from a maintained source of buoyancy and momentum) [Morton, et al. (1956), Turner (1995), Fischer, et al. (1979), Sparks (1997), Morton (1967), Socolofsky, et al. (2001)]. The entrainment hypothesis assumes that the rate of inflow of diluting, ambient fluid is proportional to the vertical velocity of the jet along its center-line. Much empirical data has been collected exploring the exact dependence of the constant of proportionality upon the various physical parameters describing the jet configuration (stratified profile, jet speed, jet fluid density). A considerable effort since the original work of Morton, et. al., has addressed the many algebraic fits for the entrainment coefficient as a function of Richardson numbers, etc. [Socolofsky, et al. (2001), Fischer, et al. (1979), Turner (1995)].

Armed with this entrainment assumption, Morton, et al., [Morton, et al. (1956)] developed the following system of nonlinear ordinary differential equations, the solution of which yields the jet (plume) profile in steady state.

\[ \frac{d(b^2w)}{dz} = 2\alpha bw \]  
\[ \frac{d(b^2w^2)}{dz} = 2gX^2b^2Q \]
\[
\frac{d(b^2 wQ)}{dz} = \left( \frac{(1 + \lambda^2) b^2 w}{\lambda^2 \bar{\rho}} \right) \frac{d\rho_0}{dz}
\]

Here, the plume parameters are the center-line vertical velocity, \( w(z) \), the plume radius, \( b(z) \), and the non-dimensional plume density is, \( Q(z) \). All are functions of the height variable, \( z \). The entrainment coefficient is \( \alpha \), which in neutrally stratified cases is empirically seen to be approximately 0.08 [Turner (1995), Fischer, et al. (1979)]. The gravitational constant is \( g \), \( \bar{\rho} \) denotes some constant reference density, and the ambient stratification is contained within the given profile \( \rho_0(z) \).

This system is based on the following assumptions. First, vertical derivatives of certain horizontally averaged, low order moments (for plume mass, momentum, and buoyancy) are simplified in terms of single point, centerline field variables. Second, radial profiles for plume vertical velocity, and buoyancy are postulated in terms of “collective variables”: \( w(z) = \hat{w}(z) f(r/(ab(z))) \) and \( Q(z) = \hat{Q}(z) f(r/(cb(z))) \). Through these, the integrals defining the moments may be directly calculated, leading to the closed system of differential equations given above. For both velocity and plume density, the functional forms are taken to be Gaussians, following empirical observations [Morton (1967), Turner (1995), Fischer, et al. (1979)]. The ratio of the velocity length scale, \( a \), to plume length scale, \( c \) is \( \lambda = c/a \) is taken to be approximately 1.2, but this must certainly vary considerably upon the mixing properties of the plume fluid with the ambient. Each of these steps involves numerous approximations, an excellent list of which may be found discussed Chapter 9 of the text by Fischer, et al., along with asymptotic solutions for limiting cases [Fischer, et al. (1979)].

The solution of these equations give a rough picture for plume shapes in the environment and typically show plumes arresting at heights below their heights of static neutral buoyancy (in the absence of any mixing) in stratified environments. A more systematic mathematical reduction of this system from the complete equations, along with a numerical simulation of the complete fluid equations for multiphase fluid flow would be valuable.

See also Atmospheric and ocean sciences; Mixing; Navier–Stokes equation; Turbulence; Vortex dominated flows
References


